1. A box contains 10 coins, of which 5 are fair and 5 are biased to land heads with probability 0.7. A coin is drawn from the box and tossed once.

   (a) (5 points) What is the chance that it will land a head?

   (b) (10 points) Suppose that the coin drawn landed a head. Given this information, what is the conditional probability that if I draw another coin from the box(without replacing the first coin), then that coin will be a fair coin ?

2. (20 points) In a test called Narco-Analysis, a "truth" serum is given to a suspect. It is known that it is 90% reliable when the person is guilty and 99% reliable when the person is innocent. In other words 10% of the guilty are judged innocent by the serum and 1% of the innocent are judged guilty. If the suspect was selected from a group of suspects of which only 5% have ever committed a crime and the serum indicates that she is guilty, what is the probability that she is innocent ?

3. (15 points) Let X be the minimum and Y be the maximum of three digits picked at random wthout replacement from $\{0, 1, \ldots, 9\}$. Find the joint distribution of X and Y.

4. (20 points)Let $X \sim \text{Uniform}(0, 1)$. Let $Z = \frac{1}{X}$. Determine the probability density function of Z.

5. Let $X_1, \ldots X_n, \ldots$ be a sequence of independent and identically distributed random variables, with $E(X_1) = 10$ and $Var(X_1) = 1$.

   (a) (5 points) State the weak law of large numbers for the above mentioned sequence X_n.

   (b) (5 points) Let $T_n = \frac{10}{n} \sum_{i=1}^{n} (30 + X_i)$. Does the weak law of large numbers provide any information on T_n ?

6. (20 points) Let X be a Exponential (10) random variable with moment generating function $M_X(t)$. Let Y be another random variable with moment generating function $M_Y(t)$. Suppose $M_X(t) = M_Y(4t)$, then

   (a) Find the relationship between X and Y.

   (b) Find the probability density function of Y.